Paper Reference(s)

6678

Edexcel GCE

Mechanics M2

Advanced Level

Friday 27 January 2012 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. A tennis ball of mass 0.1 kg is hit by a racquet. Immediately before being hit, the ball has velocity $30\mathbf{i}$ m s^{-1}. The racquet exerts an impulse of $(-2\mathbf{i} - 4\mathbf{j})$ N s on the ball. By modelling the ball as a particle, find the velocity of the ball immediately after being hit.

(4)

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2. A particle P is moving in a plane. At time t seconds, P is moving with velocity $\mathbf{v}$ m s^{-1}, where $\mathbf{v} = 2t\mathbf{i} - 3t^2\,\mathbf{j}$.

Find

(a) the speed of P when $t = 4$,

(2)

(b) the acceleration of P when $t = 4$.

(3)

Given that P is at the point with position vector $(-4\mathbf{i} + \mathbf{j})$ m when $t = 1$,

(c) find the position vector of P when $t = 4$.

(5)

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3. A cyclist and her cycle have a combined mass of 75 kg. The cyclist is cycling up a straight road inclined at 5° to the horizontal. The resistance to the motion of the cyclist from non-gravitational forces is modelled as a constant force of magnitude 20 N. At the instant when the cyclist has a speed of 12 m s^{-1}, she is decelerating at 0.2 m s^{-2}.

(a) Find the rate at which the cyclist is working at this instant.

(5)

When the cyclist passes the point A her speed is 8 m s^{-1}. At A she stops working but does not apply the brakes. She comes to rest at the point B.

The resistance to motion from non-gravitational forces is again modelled as a constant force of magnitude 20 N.

(b) Use the work-energy principle to find the distance AB.

(5)

4.

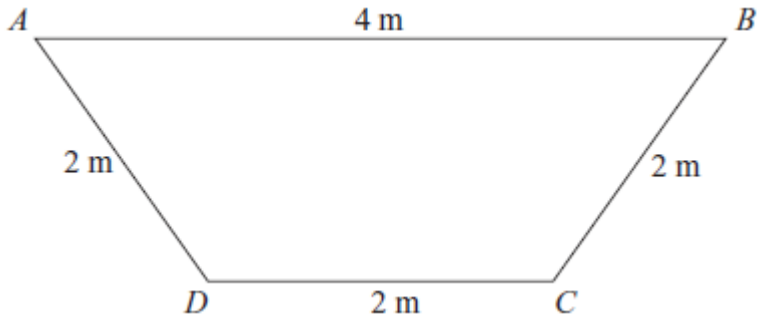

Figure 1

The trapezium $ABCD$ is a uniform lamina with $AB = 4$ m and $BC = CD = DA = 2$ m, as shown in Figure 1.

(a) Show that the centre of mass of the lamina is $\frac{4\sqrt{3}}{9}$ m from AB.

(5)

The lamina is freely suspended from D and hangs in equilibrium.

(b) Find the angle between DC and the vertical through D.

(5)

5.

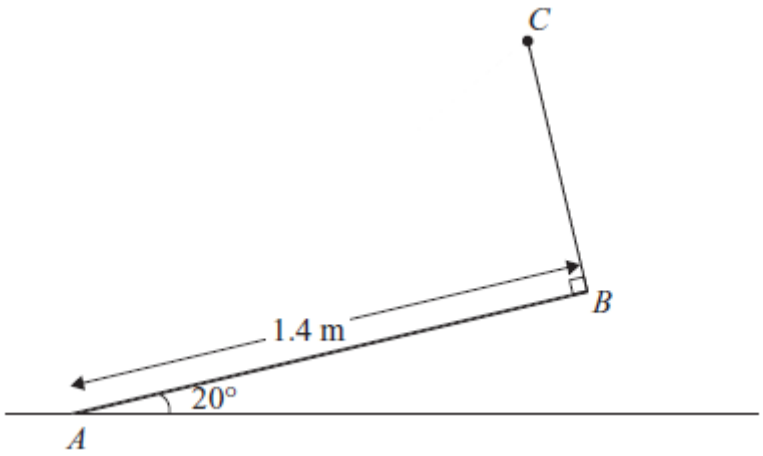

Figure 2

A uniform rod AB has mass 4 kg and length 1.4 m. The end A is resting on rough horizontal ground. A light string BC has one end attached to B and the other end attached to a fixed point C. The string is perpendicular to the rod and lies in the same vertical plane as the rod. The rod is in equilibrium, inclined at 20° to the ground, as shown in Figure 2.

(a) Find the tension in the string.

(4)

Given that the rod is about to slip,

(b) find the coefficient of friction between the rod and the ground.

(7)

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6. Three identical particles, A, B and C, lie at rest in a straight line on a smooth horizontal table with B between A and C. The mass of each particle is m. Particle A is projected towards B with speed u and collides directly with B. The coefficient of restitution between each pair of particles is $\frac{2}{3}$.

(a) Find, in terms of u,

(i) the speed of A after this collision,

(ii) the speed of B after this collision.

(7)

(b) Show that the kinetic energy lost in this collision is $\frac{5}{36}mu^2$.

(4)

After the collision between A and B, particle B collides directly with C.

(c) Find, in terms of u, the speed of C immediately after this collision between B and C.

(4)

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7. [*In this question, the unit vectors* **i** *and* **j** *are horizontal and vertical respectively*.]

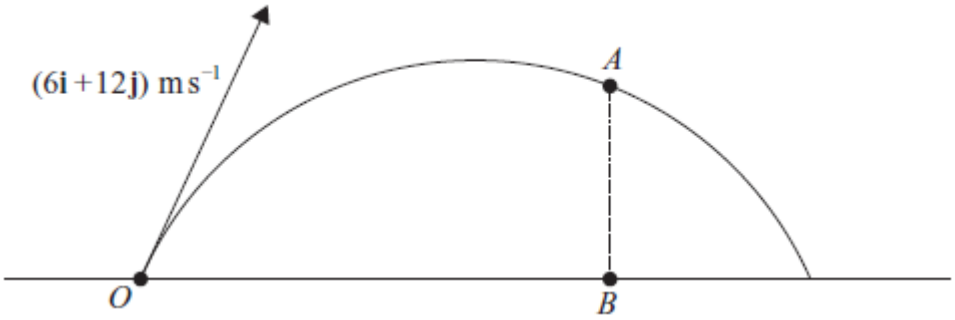


Figure 3

The point O is a fixed point on a horizontal plane. A ball is projected from O with velocity $(6\mathbf{i} + 12\mathbf{j})$ m s^{-1}, and passes through the point A at time t seconds after projection. The point B is on the horizontal plane vertically below A, as shown in Figure 3. It is given that $OB = 2AB$.

Find

(a) the value of t,

(7)

(b) the speed, V m s^{-1}, of the ball at the instant when it passes through A.

(5)

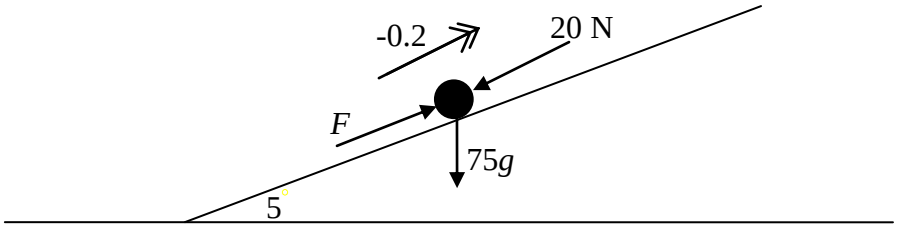
At another point C on the path the speed of the ball is also V m s^{-1}.

(c) Find the time taken for the ball to travel from O to C.

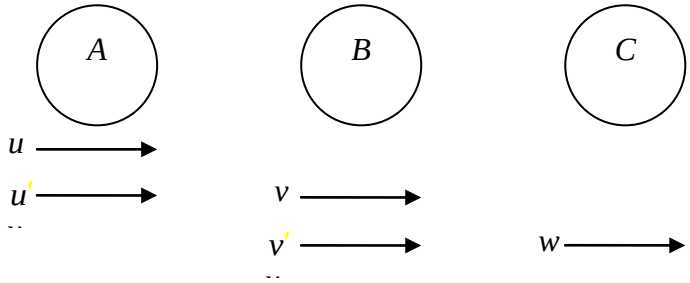
(3)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1.	Use of $m(\mathbf{v}-\mathbf{u}) = \mathbf{I}$ $0.1 \times (\mathbf{v} - 30\mathbf{i}) = -2\mathbf{i} - 4\mathbf{j}$ Solve for $\mathbf{v}$: $0.1\mathbf{v} = 3\mathbf{i} - 2\mathbf{i} - 4\mathbf{j} = \mathbf{i} - 4\mathbf{j}$ $\mathbf{v} = 10\mathbf{i} - 40\mathbf{j}$	M1 A1 M1 A1 **(4 marks)**
2. **(a)**	Speed = $\sqrt{8^2 + 48^2} = \sqrt{2368} = 48.7\,(\text{ms}^{-1})$	M1 A1 (2)
(b)	$\mathbf{a} = 2\mathbf{i} - 6t\,\mathbf{j}$ When $t = 4$, $\mathbf{a} = 2\mathbf{i} - 24\,\mathbf{j}\ (\text{ms}^{-2})$	M1 A1 A1 (3)
(c)	$\mathbf{r} = t^2\mathbf{i} - t^3\mathbf{j} + \mathbf{C}$ t = 1, $-4\mathbf{i} + \mathbf{j} = \mathbf{i} - \mathbf{j} + \mathbf{C}$, $\mathbf{C} = -5\mathbf{i} + 2\mathbf{j}$ $\mathbf{r} = (t^2 - 5)\mathbf{i} + (-t^3 + 2)\mathbf{j}$ When $t = 4$, $\mathbf{r} = (16-5)\mathbf{i} + (-64 + 2)\mathbf{j} = 11\mathbf{i} - 62\mathbf{j}$	M1 A1 M1 M1 A1 (5) **(10 marks)**
3. **(a)**	 Driving force = F Resolving parallel to the plane: $F - 20 - 75g\sin 5 = -75 \times 0.2 = -15$ $F = 5 + 75g\sin 5°$ $P = Fv$ ∴ working at $12 \times (5 + 75g\sin 5°) = 828.7.....$ ≈ 830 W	M1 A2 – lee DM1 A1 (5)
(b)	Loss in KE = gain in GPE + work done against resistance $\frac{1}{2} \times 75 \times 64 = 75 \times 9.8 \times \sin 5° d + 20d = d \times 84.059....$ $d = 28.6$ m	M1 A2 – lee DM1 A1 (5) **(10 marks)**

Question Number	Scheme	Marks
4. **(a)**	For an appropriate division of the trapezium into standard shapes with: correct ratio of masses correct distances of c.o.m. from AB e.g three equilateral triangles of height $\sqrt{3}$, mass m kg, com $\frac{\sqrt{3}}{3}$ from bases of each $3md = m\times\frac{2}{3}\times\sqrt{3} + 2\times m\times\frac{1}{3}\sqrt{3} = \frac{4\sqrt{3}}{3}m$, $d = \frac{4\sqrt{3}}{9}$ **AG**	 B1 B1 M1 A1 A1 (5)
(b)	Horizontal distance of c of m from D = 1m Vertical distance $\sqrt{3} - \frac{4\sqrt{3}}{9} = \frac{5\sqrt{3}}{9}$ (0.962….) $\tan^{-1}\frac{0.962...}{1}$ Angle = 43.9°	B1 B1 M1 A1ft A1 (5) **(10 marks)**
5. **(a)**	Taking moments about A: $4g\times 0.7\times\cos 20° = 1.4T$ T = 18.4 N	M1 A1 A1 A1 (4)
(b)	↑ $R + T\cos 20 = 4g$ $R = 4g - T\cos 20°$ → $F = T\sin 20$ $F = \mu R \Rightarrow T\sin 20° = \mu(4g - T\cos 20°)$ $\mu = \frac{T\sin 20°}{4g - T\cos 20°} = 0.29$	 M1 A1 M1 A1 M1 A1 A1 (7) **(11 marks)**

Question Number	Scheme	Marks
6. **(a)**		
	Momentum: $u = u' + v$	M1 A1
	NEL: $v - u' = eu$	M1 A1
	$2v = u(1 + \frac{2}{3}),\ v = \frac{1}{2}u \times \frac{5}{3} = \frac{5u}{6}$	M1 A1
	$u' = u - v = \frac{u}{6}$	A1
		(7)
(b)	KE lost $= \frac{1}{2}mu^2 - \left(\frac{1}{2}m \times \frac{25}{36}u^2 + \frac{1}{2}m \times \frac{1}{36}u^2\right)$ their speeds	M1
	$= \frac{1}{2}mu^2 - \left(\frac{1}{2}m \times \frac{26}{36}u^2\right)$	A2 – 1ee
	$= \frac{1}{2}mu^2 \times \frac{10}{35} = \frac{5}{36}mu^2$ **AG**	A1
		(4)
(c)	Speed of C $= \frac{1}{2}\left(\frac{1}{2}u\left(\frac{5}{3}\right)\right)\left(\frac{5}{3}\right) = \frac{1}{2} \cdot \frac{5u}{6} \cdot \frac{5}{3}, = \frac{25}{36}u$	M1 A1 M1 A1
		(4) **(15 marks)**

Question Number	Scheme	Marks
7		
(a)	**i** → distance = $6t$	B1
	j ↑ distance = $12t - \frac{1}{2}gt^2$	M1 A1
	At B, $2\left(12t - \frac{1}{2}gt^2\right) = 6t$	M1 A1
	$(24 - 6)t = gt^2$	M1
	$18 = gt$, $t = \frac{18}{g} (= 1.84\text{s})$	A1
		(7)
(b)	**i** → speed = 6	B1
	j ↑ velocity = $12 - gt = -6$	M1 A1
	∴ speed at A $= \sqrt{6^2 + 6^2} = \sqrt{72} = 6\sqrt{2} (= 8.49 \text{ ms}^{-1})$	M1 A1
		(5)
(c)	↑ speed = $12 - gt = +6$	M1 A1 ft
	$t = \frac{6}{g} (= 0.61\text{s})$	A1
		(3)
		(15 marks)